Technische Universität München **Theoretische Physik T70**
Physik Department **B. Garbrecht, Wenyuan Ai**

Theoretische Physik 2 (Elektrodynamik)

Wintersemester 2016/17

Abgabe bis Freitag, 25.11.16, 12:00 neben PH 3218.

Übungsblatt Nr. 5 Dieses Blatt wird in den Übungen vom 28.11. - 02.11.16 besprochen.

Aufgabe 1:
Method of images I — 4 Punkte

Using the method of images, discuss the problem of a point charge q inside a hollow, ground, conducting sphere of inner radius a. Find

(a) the potential inside the sphere,

(b) the induced surface-charged density on the inner surface,

(c) the magnitude and direction of the force acting on q.

(d) Is there any change in the solution if the sphere is kept at a fixed potential V? If the sphere has a total charge Q on its inner and outer surfaces?

Hint: You may use the results found in the lectures for a charge placed outside a sphere.

Aufgabe 2:
Method of images II — 4 Punkte

A straight-line charge with constant linear charge density λ (*i.e.* charge per unit length) is located perpendicular to the $x-y$ plane in the first quadrant at (x_0, y_0). The intersecting plane $x = 0$, $y \geq 0$ and $y = 0$, $x \geq 0$ are conducting boundary surfaces held at *zero* potential. Consider the potential, fields, and surfaces charges in the first quadrant.

(a) The well-known potential for an isolated line charge at (x_0, y_0) is

$$\Phi(x, y) = \lambda \ln \frac{R^2}{r^2},$$

where $r^2 = (x - x_0)^2 + (y - y_0)^2$ and R is a constant. Determine the expression for the potential of the line charge in the presence of intersecting planes. Verify explicitly that the potential and the tangential electric field vanish on the boundary surface.

(b) Determine the surface charge density σ on the plane $y = 0$, $x \geq 0$. Plot σ/λ versus x for $(x_0 = 2, y_0 = 1)$, $(x_0 = 1, y_0 = 1)$ and $(x_0 = 1, y_0 = 2)$.

(c) Show that the total charge (per unit length in z) on the plane $y = 0$, $x \geq 0$ is

$$Q_x = -\frac{2}{\pi}\lambda \tan^{-1}\left(\frac{x_0}{y_0}\right).$$

What is the total charge on the plane $x = 0$?

(d) Show that far from the origin, i.e. $\rho = \sqrt{x^2 + y^2} \gg \rho_0 = \sqrt{x_0^2 + y_0^2}$, the leading term in the potential is

$$\Phi \to \Phi_{\text{asym}} = 16\lambda \frac{(x_0 y_0)(xy)}{\rho^4}.$$

Aufgabe 3:
Boundary value problem **2 Punkte**

Let the charge free volume $V = \{\mathbf{r} : 0 < x < a,\, 0 < y < \infty,\, -\infty < z < \infty\}$ be bounded by three metal plates that are isolated from one another. The two side panels at $x = 0$ and $x = a$ are grounded, i.e. at *zero* potential. The bottom panel at $y = 0$ is kept at the constant potential ϕ_0. Due to the translational symmetry in z direction the potential $\phi(\mathbf{r})$ only depends on x and y. Verify that inside of V the solution is

$$\phi(x, y) = \frac{2\phi_0}{\pi} \arctan \frac{\sin(\pi x/a)}{\sinh(\pi y/a)} \,.$$

Hint: Show

$$\begin{aligned} \phi(0, y) &= 0, \;\text{ for } y > 0; \\ \phi(a, y) &= 0, \;\text{ for } y > 0; \\ \lim_{y \to 0^+} \phi(x, y) &= \phi_0, \;\text{ for } x \neq 0, \neq a; \\ \Delta\phi(x, y) &= 0, \;\text{ for } 0 < x < a, \; y > 0. \end{aligned}$$