Theoretische Physik 2 (Elektrodynamik)

Wintersemester 2016/17

Abgabe bis Freitag, 18.11.16, 12:00 neben PH 3218.

Übungsblatt Nr. 4 Dieses Blatt wird in den Übungen vom 21.11. - 25.11.16 besprochen.

Aufgabe 1:
The capacitance of a simple capacitor — 3 Punkte

A simple capacitor is a device formed by two insulated conductors adjacent to each other. If equal and opposite charges are placed on the conductors, there will be a certain difference of potential between them. The ratio of the magnitude of the charge on one conductor to the magnitude of the potential difference is called the capacitance C (measured in centimeters in Gaußian units). Using Gauß' law, calculate the capacitance of

(a) two large, flat, conducting sheets of area A, separated by a small distance d;

(b) two concentric conducting spheres with radii a, b ($b > a$);

(c) two concentric conducting cylinders of length L, large compared to their radii a, b ($b > a$).

(d) What is the inner diameter of the outer conductor in an air-filled coaxial cable whose center conductor is a cylindrical wire of diameter 0.1 cm and whose capacitance per centimeter is 2.7? 0.27?

Aufgabe 2:
The electrostatic energy of a simple capacitor — 3 Punkte

(a) For the three capacitor geometries in Problem 1, calculate the total electrostatic energy and express it in alternatively terms of the equal and opposite charges Q and $-Q$ placed on the conductors *and* the potential difference between them.

Hint: Show that

$$W = \frac{1}{8\pi}\int |\mathbf{E}|^2\, d^3x = \frac{1}{2}CV^2$$

(b) Let $d = b - a$. Sketch the energy density of the electrostatic field in each case as a function of the appropriate linear coordinate assuming the energy density are equal at $r = a$.

Aufgabe 3:
Thomson's theorem — 4 Punkte

Background: In this problem, we aim to prove the *Thomson's theorem*: If a number of surfaces are fixed in position and a given total charge is placed on each surface, then the electrostatic energy in the region bounded by the surfaces is an absolute minimum when the charges are placed so that every surface is an equipotential, as happens when they are conductors.

(a) Consider n volumes V_i bounded by surfaces S_i with total charges q_i ($i = 1, 2, ..., n$) on the surface. Note that because the volumes are not necessarily simply connected, the surfaces

S_i may consist of several disjoint areas. Denote the total volume and area by $V = \sum_{i=1}^{n} V_i$ and $\partial V = \sum_{i=1}^{n} \partial V_i$. Let's consider two distributions of the charges $\rho_1 = \rho_{\text{surface}} + \rho_{\text{bulk}}$ and $\rho_2 = \rho'_{\text{surface}} + \rho_{\text{bulk}}$, i.e. ρ_{bulk} is fixed). Under the charge distribution ρ_1, every surface is an equipotential. ρ_2 is an arbitrary different one (under the constraint of total charge on the surface). Let the field configurations corresponding to ρ_1 and ρ_2 be given by ϕ, $\mathbf{E}$ and ϕ', $\mathbf{E}'$, respectively. Show that in the regions bounded by the surfaces

$$\nabla \cdot (\mathbf{E}' - \mathbf{E}) = 0$$

and on the surfaces

$$\int_{\partial V_i} (\mathbf{E}' - \mathbf{E}) \cdot \mathbf{n} \, da = 0.$$

(b) Let W and W' be the electrostatic energy in the regions bounded by the surfaces under distributions ρ_1 and ρ_2. Prove $W' - W > 0$.

Hint: Make use of

$$E'^2 - E^2 = (\mathbf{E}' - \mathbf{E})^2 - 2\mathbf{E} \cdot (\mathbf{E}' - \mathbf{E}),$$

where the first term is manifestly positive. For the second term, show that

$$\int_V \mathbf{E} \cdot (\mathbf{E}' - \mathbf{E}) \, d^3x = - \int_{\partial V} \phi(\mathbf{E}' - \mathbf{E})$$

where you can use one of the result from part (a). Make use of the other result from part (a) to complete the proof.