Theoretische Physik 2 (Elektrodynamik)

Wintersemester 2016/17

Abgabe bis Freitag, 11.11.16, 12:00 neben PH 3218.

Übungsblatt Nr. 3 Dieses Blatt wird in den Übungen vom 14.11. - 18.11.16 besprochen.

Aufgabe 1:
The normal derivative of the electric field at the surface of a curved charged conductor 3 Punkte

Use Gauß' theorem to prove that at the surface of a curved charged conductor, the normal derivative of the normal component of the electric field is given by

$$\frac{1}{E}\frac{\partial E}{\partial n} = -(\frac{1}{R_1} + \frac{1}{R_2}),$$

where R_1, R_2 are the principal radii of curvature at the point of interest at the surface.
Hint: Consider a surface element $d\sigma = R_1 d\theta_1 R_2 d\theta_2 \equiv R_1 R_2 d\Omega$, as well as another surface element, where the boundaries are shifted by an amount ε along the normal vectors. Apply Gauß' theorem to the volume between these surfaces. It may be helpful to consider the simplified case a cylinder first, *e.g.* by setting one of the two R_i to be infinity such that the surface of the conductor looks like locally the surface of a cylinder. Is the surface given as a height function on the two-dimensional plane, the inverse of the radii of principal curvature are given by the eigenvalues of the Hessian of the height function. A sketch of the two surface elements may also be helpful.

Aufgabe 2:
The transformation of delta function 2 Punkte

(a)Let $\delta(\mathbf{x} - \mathbf{x}')$ be the 3-dimensional delta function in the orthogonal coordinate system $\{xyz\}$. Consider a general orthogonal coordinate system specified by the surfaces $u = constant$, $v = constant$, $w = constant$ with length elements du/U, dv/V, dw/W in the three perpendicular directions (*i.e.* it is generated through a rotation and rescaling of the individual axes). Show that

$$\delta(\mathbf{x} - \mathbf{x}') = \delta(u - u')\delta(v - v')\delta(w - w') \cdot UVW.$$

(b)Given that the 3-dimensional delta function $\delta(\mathbf{x})$ can be taken as the improper limit as $\alpha \to 0$ of the Gaussian function

$$D(\alpha; x, y, z) = (2\pi)^{-3/2}\alpha^{-3} \exp\left[-\frac{1}{2\alpha^2}\left(x^2 + y^2 + z^2\right)\right],$$

prove the result of (a) by taking direct calculations of this function.

Aufgabe 3:
delta function **2 Punkte**

Do the integrals below:

$$
\begin{aligned}
&(a).\ \int_0^\infty x^2\delta(x^2-3x+2)dx;\\
&(b).\ \int_0^\infty \ln x\,\delta'(x-2)dx;\\
&(c).\ \int_0^\pi \sin^3\theta\,\delta\left(\cos\theta-\cos\frac{\pi}{3}\right)\,d\theta.
\end{aligned}
$$

Hint: For a function $f(x)$ with roots x_i all simple, we have

$$\delta(f(x)) = \sum_i \frac{\delta(x-x_i)}{|f'(x_i)|}.$$

Aufgabe 4:
The flux of the vector field **3 Punkte**

Consider the field $\mathbf{V}(\mathbf{x}) = \frac{c}{\rho}\,\mathbf{e}_\rho$, where c is constant and $\mathbf{e}_\rho = (\cos\varphi, \sin\varphi, 0)^t$. (cf. Sheet 2, Problem 3)

(a) For a cylinder of length l and radius R that is symmetric around the z axis, calculate the flux of $\mathbf{V}$ through its surface, i.e.

$$F = \int_{\partial\,\text{cylinder}} d\mathbf{a}\cdot\mathbf{V}(\mathbf{x}).$$

(b) Calculate $\nabla\cdot\mathbf{V}(\mathbf{x})$ including the singular part by use of Gauß' law.

(c) An infinitely long straight wire (at $z=0$) bears the charge κ per unit length. Find the electric field using (i) the result from part (b) and (ii) the general formula valid for a 3-dimensional distribution.

Hint: You may need the formula:

$$\int dz \frac{1}{(\sqrt{a^2+z^2})^3} = \frac{z}{a^2\sqrt{a^2+z^2}}.$$