Technische Universität München
Physik Department

Theoretische Physik T70
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Theoretische Physik 2 (Elektrodynamik)

Wintersemester 2016/17

Abgabe bis Freitag, 4.11.16, 12:00 neben PH 3218.

Übungsblatt Nr. 2

Dieses Blatt wird in den Übungen vom 7.11. - 11.11.16 besprochen.

Aufgabe 1:
Identities for vector field — 3 Punkte

Given two smooth vector fields $\mathbf{A}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$, verify the following identities:

(a) $\mathrm{div}(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\mathrm{rot}\mathbf{A} - \mathbf{A}\cdot\mathrm{rot}\mathbf{B}$,

(b) $\mathrm{rot}(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,\mathrm{div}\mathbf{B} - \mathbf{B}\,\mathrm{div}\mathbf{A} + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$,

Hint: Use the rule $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$ for the double product. Make sure that the Nabla operator operates on all the fields from the left.

(c) $\mathrm{grad}(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times\mathrm{rot}\mathbf{B} + \mathbf{B}\times\mathrm{rot}\mathbf{A} + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

Hint: Use the rule for the double product to reformulate $\mathbf{A}\times\mathrm{rot}\mathbf{B} + \mathbf{B}\times\mathrm{rot}\mathbf{A}$.

Aufgabe 2:
Stokes's theorem — 3 Punkte

Verify Stokes's theorem for the vector field

$$\mathbf{V}(\mathbf{r}) = \left(\frac{4x}{3} - 2y\right)\mathbf{e}_x + (3y - x)\mathbf{e}_y$$

on the elliptical plane $(x/3)^2 + (y/2)^2 \leq 1$, $z = 0$.

Aufgabe 3:
Source free — 2 Punkte

Consider the cylindrical vector field

$$\mathbf{V}(\mathbf{r}) = v(\rho)\mathbf{e}_\rho, \quad \rho = \sqrt{x^2+y^2} \neq 0,\ \ \mathbf{e}_\rho = (\cos\varphi, \sin\varphi, 0)^t.$$

Find $v(\rho)$ such that $\mathbf{V}(\mathbf{r})$ is divergence free .

Hint: Use Cartesian coordinates.

Aufgabe 4:
Mean value of the potential theory — 2 Punkte

Prove the mean value theorem: For charge free space ($\Delta\Phi = 0$) the value of the electrostatic potential is equal to the average of the potential over the surface of any sphere centred on that point.

Hint: Start with the identity for the mean value

$$\frac{1}{4\pi R^2} \iint_{|\mathbf{r}'-\mathbf{r}|=R} dF' \, \Phi(\mathbf{r}') = \frac{1}{4\pi} \iint_{S^2} d\Omega \, \Phi(\mathbf{r} + \mathbf{n}R),$$

where $\mathbf{n}$ is the unit outward-pointing normal vector on a sphere of radius R. Use the Gauß theorem and $\nabla\cdot(\nabla\Phi) = \Delta\Phi = 0$ to show that the derivative of the right-hand-side expression with respect to R vanishes.