Theoretische Physik 2 (Elektrodynamik)

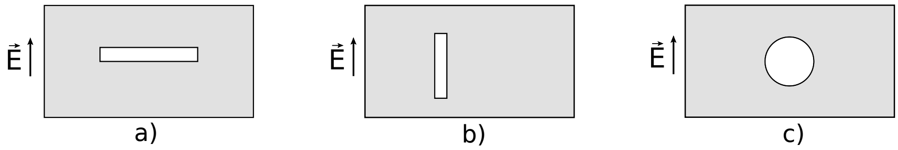
Wintersemester 2016/17

Abgabe bis Freitag, 20.01.16, 12:00 neben PH 3218.

Übungsblatt Nr. 10 Dieses Blatt wird in den Übungen vom 23.01. - 27.01.16 besprochen.

Aufgabe 1:
Cavities within a dielectric material — 5 Punkte

Let there be an asymptotically homogeneous field $\mathbf{E}$ within a dielectric material with permittivity $\varepsilon > 1$. Calculate the field $\mathbf{E}_{\text{in}}$ inside the following cavities:



a) a narrow slit (but with extended side area) perpendicular to $\mathbf{E}$,
b) a narrow slit (but with extended side area) parallel to $\mathbf{E}$,
c) a spherical cavity of radius R.
Hint: For part c), make use of the results for the dielectric sphere (as discussed in the lectures) with modifications pertaining to the modified boundary conditions (inside $\leftrightarrow$ outside).

Aufgabe 2:
Radiation of a point charge on a circular orbit — 5 Punkte

Point charges on circular orbits appear *e.g.* in accelerators or in astrophysical magentic fields. Their radiation limits the ultimate energy up to which the particles can be accelerated. Moreover, electrons on circular orbits lose energy by radiation. Therefore, such a classical model of atomic physics would not allow for stable energy levels, which had been one of the main problems of classical physics that has led to the discovery of quantum mechanics.

a) Let a particle of charge q be moving at an angular velocity ω on a circle of radius R. (Assume $R \ll c/\omega$, such that the speed is non-relativistic.) This leads to the time-dependent charge-density

$$\rho(\mathbf{r}, t) = q\,\delta(x - R\cos\omega t)\,\delta(y - R\sin\omega t)\,\delta(z)\,.$$

Calculate the pertaining dipole moment $\mathbf{p}(t)$ and express this through a complex vector $\mathbf{p}$ satisfying the relation $\mathbf{p}(t) = \mathrm{Re}[\mathbf{p}\exp(-i\omega t)]$. Make use of the leading order result for the radiation in the far field zone and derive the angular distribution of the differential radiation power $dP/d\Omega$ and integrate this to obtain the total power of the radiation. (2.5 Punkte)

b) Following classical mechanics and the Coulomb force, an elctron would move on a stable circular orbit around the proton. Express the angular frequency ω and the energy (sum of kinetic and potential contributions) of the electron as a function of the radius r. The loss of radiation power would then lead to an orbital radius $r(t)$ that decreases with time. Set up a differential equation (*Hint*: It is of the form $\dot{r}r^2 = \text{const.}$) and integrate it with the initial condition $r(0) = a_{\text{B}} \simeq 5.29 \times 10^{-9}$cm (Bohr radius). After what time τ does the electron reach the nucleus? (*Hint*: The answer is $\tau \simeq 1.56 \times 10^{-11}$s.) (2.5 Punkte)