

Theoretische Physik 2 (Elektrodynamik)

Wintersemester 2016/17

Abgabe bis Freitag, 28.10.16, 12:00 neben PH 3218.

Übungsblatt Nr. 1

Dieses Blatt wird in den Übungen vom 31.10. - 04.11.16 besprochen.

Aufgabe 1:

The symbol ε_{ijk}

3 Punkte

(a) Express ε_{ijk} in terms of a determinant.

Hint: Use Leibniz' formula:

$$\det M = \sum_{\sigma \in \mathcal{S}_n} \text{sign } \sigma \prod_{i=1}^3 M_{i\sigma(i)}.$$

(b) Show that

$$\varepsilon_{klm}\varepsilon_{pqn} = \begin{vmatrix} \delta_{kp} & \delta_{kq} & \delta_{kn} \\ \delta_{lp} & \delta_{lq} & \delta_{ln} \\ \delta_{mp} & \delta_{mq} & \delta_{mn} \end{vmatrix}.$$

Hint: Use matrix multiplication.

(c) Show that $\varepsilon_{klm}\varepsilon_{pqm} = \delta_{kp}\delta_{lq} - \delta_{kq}\delta_{lp}$.

Hint: Recall the Laplace's formula

$$\det M = \sum_{j=1}^3 (-1)^{i+j} M_{ij} \det \bar{M}_{ij},$$

where \bar{M}_{ij} is the matrix M with line i and column j deleted.

Aufgabe 2:

Vector analysis

3 Punkte

Let \mathbf{M} be a constant vector. Show that

$$\nabla \times \left(\frac{\mathbf{M} \times \mathbf{x}}{|\mathbf{x}|^3} \right) = \frac{8\pi}{3} \mathbf{M} \delta^3(\mathbf{x}) - \frac{\mathbf{M}}{|\mathbf{x}|^3} + \frac{3\mathbf{M} \cdot \mathbf{x}}{|\mathbf{x}|^5} \mathbf{x}.$$

Aufgabe 3:

Cauchy principal value

2 Punkte

(a) Show that

$$\lim_{\epsilon \rightarrow 0} \frac{\epsilon}{(x-a)^2 + \epsilon^2} = \pi \delta(x-a).$$

(b) Show that under the integral $\int_{-\infty}^{\infty} dx$

$$\lim_{\epsilon \rightarrow 0^+} \frac{1}{(x-a) \mp i\epsilon} = \pm i\pi\delta(x-a) + \mathcal{P} \frac{1}{x-a},$$

where $\mathcal{P}(\dots)$ is the Cauchy principal value.

Aufgabe 4:

Vector analysis and Gauß theorem

2 Punkte

Show that

$$\int_V d^3x \mathbf{u}(\mathbf{x}) \cdot (\nabla \times \mathbf{v}(\mathbf{x})) = \int_V d^3x \mathbf{v}(\mathbf{x}) \cdot (\nabla \times \mathbf{u}(\mathbf{x})) - \int_{\partial V} d\mathbf{a} (\mathbf{u}(\mathbf{x}) \times \mathbf{v}(\mathbf{x})).$$