# Exercises for CMB and LSS

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due until Thursday, June 23, 2016 in class

**SS 2016**
**Sheet 4**

**Problem 1.**
The full general relativistic expression for the energy momentum tensor is

$$T^\mu{}_\nu(\overrightarrow{x},t)\Big|_{\text{species i}} = g_i \int \frac{dP_1 dP_2 dP_3}{(2\pi)^3}(-\det[g_{\alpha\beta}])^{(-1/2)}\frac{P^\mu P_\nu}{P^0} f_i(\overrightarrow{x},\overrightarrow{p},t),$$

where $P_\mu$ is the four-momentum and $g_i$ is the number of spin states for species i. It holds even in the presence of metric perturbations. Use this expression to show that, with scalar perturbations to the metric, the phase space integral for the time-time component reduces to the form

$$T^0{}_0(\overrightarrow{x},t) = -\sum_{\text{all species i}} g_i \int \frac{d^3p}{(2\pi)^3} E_i(p) f_i(\overrightarrow{x},\overrightarrow{p},t).$$

And show that the contribution from species $\alpha$ to $T^0_i$ is

$$T^0{}_i = g_\alpha a \int \frac{d^3p}{(2\pi)^3} p_i f_\alpha(\overrightarrow{x},\overrightarrow{p},t).$$

Note the extra factor of $a$.

**Problem 2.**
Compute the time-space component of the Einstein tensor. Show that, in Fourier space,

$$G^0{}_i = 2ik_i\left(\frac{\dot{\Phi}}{a} - H\Psi\right).$$

Combine with the energy-momentum tensor derived in problem 1 to show that

$$\dot{\Phi} - aH\Psi = \frac{4\pi G a^2}{ik}\left[\rho_{dm} v + \rho_b v_b - 4i\rho_\gamma \Theta_1 - 4i\rho_\nu \mathcal{N}_1\right].$$

The time-space component of Einstein's equations adds no new information once we already have the two equations derived in the text. Deciding which two to use is a matter of convenience.