Exercises for CMB and LSS

Lecturer: Björn Garbrecht
Sheets by Wenyuan Ai
due until Tuesday, June 7, 2016 in class

SS 2016
Sheet 3

Problem 1.
According to the perturbations to the photo distribution function

$$f(\vec{x}, p, \vec{p}, t) = \left[\exp\left\{ \frac{p}{T(t)[1 + \Theta(\vec{x}, \vec{p}, t)]} \right\} - 1 \right]^{-1},$$

show that, if Θ depends only on μ, the cosine of the angle between $\hat{k}(\equiv \hat{z}$ here) and $\hat{p}$, then $T^1{}_1 - T^2{}_2$ vanishes. This is yet another aspect of the decomposition theorem: the terms Θ that source the scalar perturbations (and are sourced by them) do not affect tensor perturbations. Actually, the tensor perturbations will induce anisotropies of the form

$$\Theta(\mu, \phi) = (1 - \mu^2) \cos(2\phi) \Theta_+(\mu)$$

for those perturbations generated by h_+ and a similar form for $h_\times$ with the *cos* replaced by *sin*. These, however, have a negligible impact on the evolution of the gravity waves, so we are justified in setting the right-hand side of

$$\ddot{h}_+ + 2\frac{\dot{a}}{a}\dot{h}_+ + k^2 h_+ = 0$$

to zero.

Problem 2.
Consider tensor perturbations to the metric. These do not perturb $g_{00}(= -1)$ or $g_{0i}(= 0)$. However, the spatial part of the metric is now

$$g_{ij} = a^2 \begin{pmatrix} 1 + h_+ & h_\times & 0 \\ h_\times & 1 - h_+ & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Derive the equation for the photon distribution function in the presence of tensor perturbations. Unlike scalar perturbations, tensor perturbations induce an azimuthal dependence in θ_l, so decompose the anisotropy due to tensor into

$$\Theta^T(k,\mu,\phi) = \Theta^T_+(k,\mu)(1-\mu^2)\cos(2\phi) + \Theta^T_\times(k,\mu)(1-\mu^2)\sin(2\phi).$$

Show that both the + and the $\times$ component satisfy

$$\frac{d\Theta^T_i}{d\eta} + ik\mu\Theta^T_i + \frac{1}{2}\frac{dh_i}{d\eta} = \dot{\tau}\left[\Theta^T_i - \frac{1}{10}\Theta^T_{i,0} - \frac{1}{7}\Theta^T_{i,2} - \frac{3}{70}\Theta^T_{i,4}\right]$$

where i stands for either + or $\times$, and the moments are defined as $\Theta_l \equiv \frac{1}{(-i)^l}\int_{-1}^{1}\frac{d\mu}{2}\mathcal{P}_l(\mu)\Theta(\mu)$.

I have made the solutions to previous problems into pdf form. In case you need them, please contact me via wenyuanai@hotmail.com