Exercises for CMB and LSS

SS 2016
Sheet 2

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due until Tuesday, May 10, 2016 in class

Problem 1.
The ESA's Planck satellite, dedicated to studying the early universe, was launched on May 2009 and has been surveying the microwave and submillimetre sky since August 2009. A major goal of the Planck experiment is to determine with great precision the key cosmological parameters describing our Universe. The results were released in March 2013. Find out cosmological parameters for the base ΛCDM model in paper arXiv:1303.5076, and use them to compute following quantities.

- The ratio of baryon number to photon number $\frac{n_B}{n_\gamma}$.
- The ratio of baryon number to the entropy $\frac{n_B}{s}$ (baryon number of the universe).
- The red shift Z_{eq}, temperature T_{eq}, and time t_{eq} of matter-radiation equality phase at the early stage of universe.

Problem 2.
Start from the zero-order collisionless Boltzmann equation for photons

$$\left.\frac{df}{dt}\right|_{\text{zero order}} = \frac{\partial f^{(0)}}{\partial t} - Hp\frac{\partial f^{(0)}}{\partial p} = 0,$$

where $p^2 \equiv g_{ij}P^iP^j$. Integrate this equation over all momenta to show that the number density falls off as a^3.

Problem 3.
The Klein-Nishina formula for the Compton scattering has the form

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right],$$

where ω, ω' are the frequencies of photos before and after the scattering and θ is the scattering angle. Derive the Thomson cross section in the non-relativistic limit $\omega \to 0$.
Hint: Use the Compton's formula for the shift in the photo frequency. For those who are interested in the derivation of Klein-Nishina formula, see, e.g. Peskin & Schroeder pp. 158-163.