Exercises for CMB and LSS

Lecturer: Björn Garbrecht
Sheets by Wenyuan Ai
to be discussed on Tuesday, April 26th, 2016 in class

SS 2016
Sheet 1

Problem 1.
Convert the following quantities by inserting the appropriate factors of c, $\hbar$, and k_B:

- CMB temperature $T_0 = 2.725K \to eV$
- Photon energy density $\rho_\gamma = \pi^2 T_0^4/15 \to eV^4$ and $g\,cm^{-3}$
- Typical horizon scale $1/H_0 \to cm$
- Planck mass $m_{Pl} \equiv 1.2 \times 10^{19} GeV \to K,\ cm^{-1},\ sec^{-1}$

Problem 2.
From the lecture note, one knows that Christoffel symbols can be expressed as

$$\Gamma^\kappa_{\mu\nu} = \frac{\partial x^\kappa}{\partial \xi^\alpha}\frac{\partial^2 \xi^\alpha}{\partial x^\mu \partial x^\nu}, \qquad \text{and} \quad g_{\mu\nu} = \eta_{\mu\nu}\frac{d\xi^\alpha}{dx^\mu}\frac{d\xi^\beta}{dx^\nu},$$

where ξ^α and x^μ are components of a local Minkowski and a global coordinate system respectively. Show that this expression is equivalent to the formula

$$\Gamma^\sigma_{\mu\nu} = \frac{g^{\sigma\rho}}{2}\left(\frac{\partial g_{\nu\rho}}{\partial x^\mu} + \frac{\partial g_{\mu\rho}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho}\right).$$

Problem 3.
Modern cosmological theories can exhibit horizons of two different types, which limit the distances at which past events can be observed or at which it will ever be possible to observe future events. These are called by Rindler particle horizons and event horizons, respectively.
Let we set the big bang started at a time $t = 0$ and the space is flat ($\kappa = 0$).
(a) If the universe is dominated by radiation, compute the greatest value of comoving distance d_{max}, which an observer at time t_0 is able to receive signal from the big bang. (radius of particle horizon)
(b) If the universe is dominated by cosmological constant Λ, it will be expanding forever. Computer the largest comoving distance d_{max} which the

events occurring at time t_0 that we can ever observe. (radius of event horizon)
Hint: one can adopt a spherical coordinate system for space components, and consider a light ray coming to the observer along the radial direction.

Problem 4.

Let's take the data of the energy densities for matter, radiation and cosmological constant (dark energy in ΛCDM model) to be $\rho_{0M} = 0.315$, $\rho_{0\Lambda} = 0.685$ and $\rho_{0R} = \frac{\rho_{0M}}{3401}$ respectively.
(a) Please make a plot of the energy densities as functions of the scale factor.
(b) Please make another plot of the energy density propotions as functions of the scale factor and explain how the picture indicates the coincidence problem in cosmology.
Hint: It may be better for you to take a log-log plot for (a) and a log-linear plot for (b). The plots can be found in the lecture notes, but the point of this exercise is that you create your own version, using e.g. Mathematica or you favourite computer tool, based on the correct cosmological input parameters.