# GENERAL RELATIVITY WS 2017/2018
Technische Universität München
December 1, 2017

## Exercise Sheet 7*

The solutions to the following problem set should be handed in by the 8st of December at 8:30 a.m. at the postbox next to PH 3218.

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1. For $\mathbb{R}^3$ equipped with the Euclidean metric, consider the following vector fields:

$$A = \frac{y-x}{r}\frac{\partial}{\partial x} - \frac{x+y}{r}\frac{\partial}{\partial y} \tag{1}$$

$$B = xy\frac{\partial}{\partial x} - y^2\frac{\partial}{\partial y}. \tag{2}$$

   Compute the integral curves for these fields together with $C = \mathcal{L}_A B$.

2. Prove the following identities for a Killing vector $K^\mu$:

$$\nabla_\mu \nabla_\sigma K^\rho = R^\rho{}_{\sigma\mu\nu} K^\nu \tag{3}$$

$$K^\lambda \nabla_\lambda R = 0 \tag{4}$$

3. Find a complete set of Killing vectors for the following spaces:

   (a) Minkowski space, $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$.

   (b) A space-time with coordinates $u, v, x, y$ with the metric:

$$ds^2 = -(dudv + dvdu) + a^2(u)dx^2 + b^2(u)dy^2 \tag{5}$$

   where $a, b$ are some smooth function of $u$. This space-time is associated to gravitational waves. (There are in total five of them)

4. Show that the Weyl tensor $C^\rho{}_{\mu\nu\lambda}$ is conformal invariant.

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