GENERAL RELATIVITY WS 2017/2018

Technische Universität München

November 20, 2017

Exercise Sheet 5*

The solutions to the following problem set should be handed in by the 24th of November at 8:30 a.m. at the postbox next to PH 3218.

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1. Prove that all geodesics on S^2 are great circles (meaning circles which are contained in a plane that passes through the origin, e.g. the equator).

2. Given a diagonal metric $g_{\mu\nu}$, prove that the Christoffels symbols, can be computed in the following way, assume $\mu \neq \nu \neq \lambda$ and that there is no summation convention:

$$\Gamma^{\lambda}_{\mu\nu} = 0 \tag{1}$$

$$\Gamma^{\lambda}_{\mu\mu} = -\frac{1}{2g_{\lambda\lambda}}\partial_{\lambda} g_{\mu\mu} \tag{2}$$

$$\Gamma^{\lambda}_{\mu\lambda} = \partial_{\mu}\left(\ln\sqrt{|g_{\lambda\lambda}|}\right) \tag{3}$$

$$\Gamma^{\lambda}_{\lambda\lambda} = \partial_{\lambda}\left(\ln\sqrt{|g_{\lambda\lambda}|}\right) \tag{4}$$

3. The metric close to the Earth's surface can be accurately described by:

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{5}$$

with

$$\Phi = -\frac{GM}{r} \tag{6}$$

where G is Newton's constant and M is the Earth's mass.

   (a) Imagine two clocks at distances R_1 and R_2 from the center of the Earth, with $R_2 > R_1$. Which clock ticks faster?

   (b) Solve the geodesic equation for an orbit constrained to the equator, $\theta = \pi/2$. What is the angular velocity $d\phi/dt$?

   (c) During one orbit of such geodesic at a radius R, how much proper time elapses? (assume $\Phi \ll 1$) Make R equal to the radius of the earth and get an answer in seconds. Compare this number with the proper time elapsed for the stationary clock in item 3a.

4. Generalize the result for the curvature of a sphere seen in the lecture, to the n-dimensional case.

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