GENERAL RELATIVITY WS 2017/2018

Technische Universität München

January 17, 2018

Exercise Sheet 11*

The solutions to the following problem set should be handed in by Monday the 22th of January at 8:30 a.m. at the postbox next to PH 3218.

Comoving coordinates and gravitational collapse of a perfect fluid. Part 2.

1. Using the field equations and the energy conservation equation derived in part 1 last week,

$$-4\pi G\rho = \frac{1}{U}\left[\frac{V''}{V} - \frac{V'^2}{2V^2} - \frac{U'V'}{2UV}\right] - \frac{\ddot{U}}{2U} + \frac{\dot{U}^2}{4U^2} - \frac{\dot{U}\dot{V}}{2UV}, \tag{1}$$

$$-4\pi G\rho = -\frac{1}{V} + \frac{1}{U}\left[\frac{V''}{2V} - \frac{U'V'}{4UV}\right] - \frac{\ddot{V}}{2V} - \frac{\dot{V}\dot{U}}{4UV}, \tag{2}$$

$$-4\pi G\rho = \frac{\ddot{U}}{2U} + \frac{\ddot{V}}{V} - \frac{\dot{U}^2}{4U^2} - \frac{\dot{V}^2}{2V^2}, \tag{3}$$

$$0 = \frac{\dot{V}'}{V} - \frac{V'\dot{V}}{2V^2} - \frac{\dot{U}V'}{2UV}, \tag{4}$$

   (a) assuming the density function is position independent, solve for the metric components by factorizing U and V in a product of a functions which are t- and r- dependent (Hint: to simplify things more, write the t dependent function as a square). Rescale the r coordinate conveniently to arrive to an isotropic and homogeneous form for the metric:

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right) \tag{5}$$

   (b) impose that the spatial part of the metric is 1 when the coordinate time is 0 and that the contraction begins from rest. (Hint: parametrize the solution with a cycloid) Express the density, ρ, in terms of initial data and continue to find whether the spatial part of the metric vanishes for some finite t, meaning the body has collapsed.

2. We know the exterior solution is fixed by Birkhoff's theorem so that it can be cast in the form:

$$ds^2 = -\left(1 - \frac{2M}{\bar{r}}\right)d\bar{t}^2 + \left(1 - \frac{2M}{\bar{r}}\right)^{-1} d\bar{r}^2 + \bar{r}^2 d\bar{\theta}^2 + \bar{r}^2\sin^2\bar{\theta}d\bar{\phi}^2 \tag{6}$$

   Our task now is to paste the solutions for both regions so that they match at the surface of the body, given by the comoving radius $r = a =$ constant.

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(a) Bring the solution in the interior into the barred coordinates, to do that determine the constants k,M and the relation between $\bar{r}$,$\bar{t}$ and r and t. You don't need to find an expression for the metric itself since it doesn't have an analytic expression. (Hint: Scale time so that you remove any cross-terms, make a the integrating constant for the expression for $\bar{t}$.)

(b) How much time will it take the collapse to happen as seen for a coordinate time observer at $\bar{t} = \bar{t}^*$.

(c) Compute the redshift $z = \lambda' - \lambda_0/\lambda_0$ for an observer in the outer region at a radius $\bar{r} = \bar{r}'$. How does the redshift behave for times close to the beginning of the collapse? How does it behave for longer times, "once it has collapse entirely".