GENERAL RELATIVITY WS 2017/2018

Technische Universität München

January 7, 2018

Exercise Sheet 10*

The solutions to the following problem set should be handed in by the 14th of January at 8:30 a.m. at the postbox next to PH 3218.

Comoving coordinates and gravitational collapse of a perfect fluid. Part 1.

1. Consider a metric which is spherically symmetric but otherwise as general as possible. The objective is to compute the Riemman tensor components in coordinates that are suited to study the collapse of a spherical body.

   (a) First choose a comoving set of coordinates, meaning a set of coordinates for which the tt component of the metric is equal to -1. And derive a condition on the ti components of the metric by demanding that points fixed in space should be in "free" fall, i.e. curves with $\vec{x}$ =constant and $t = \tau$ follow geodesics.

   (b) Prove that the time coordinate can be shifted by a function f as to eliminate the ti components of the metric and arrive to a metric of the form:

   $$ds^2 = -dt^2 + U(r,t)dr^2 + V(r,t)(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

   where V and U are unspecified functions for the time being.

   (c) Compute the Christoffel symbols and Ricci tensor components in these coordinates.

2. Using the energy-momentum tensor for a perfect fluid with negligible pressure,

   (a) Obtain the energy conservation equation.

   (b) Use the results of problem 1 and the four velocity of the fluid in comoving coordinates to arrive at the field equations for this region of the space-time (namely the interior of a collapsing body)†

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†There is no need to solve the equations for now, we will deal with that in the next sheet