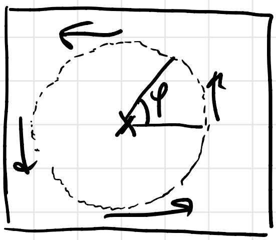
Quantum vortices in superfluids

θ – phase of the macroscopic complex order parameter, thus it does not have to be single-valued (only $e^{i\theta}$ is single-valued)

Vortex is topological defect line in a superfluid around which the phase θ winds by an integer multiple of 2π.



Axisymmetric vortex, $k \in \mathbb{N}$

$$\theta_k = k\varphi = k \arctan\frac{y}{x}$$

$$v_i = \frac{\hbar}{M} \nabla_i \theta$$

$$v_\varphi = \frac{\hbar k}{M r}$$

The circulation is quantized around the vortex

$$\oint_\ell d\ell \, v_\varphi = 2\pi \frac{\hbar}{M} k$$

1) Vortex excitation is not local, but global, since we have to change the phase θ everywhere.

2) Vortices are high-energy excitations in superfluids

$$E_k/L = \int d^2x \, \frac{\rho_s v_s^2}{2} = \frac{\rho_s}{2} \int dr \, r \, d\varphi \left(\frac{\hbar k}{M r}\right)^2$$

$$= \frac{\hbar^2}{M} n_s \pi k^2 \ln\left(R/\xi\right)$$

R – site of the container

ξ – coherence length – an intrinsic length scale of a SF that determines how the SF responds to external potentials and impurities:

fermionic SF $\quad \xi_{BCS} \sim k_F/\Delta$

bosonic SF $\quad \xi_{BEC} \sim 1/\sqrt{n_0 a}$

we conclude $E_k/L \to \infty$ if $R/\xi \to \infty$

3) $k > 1$ vortices are unstable

$E_k \sim k^2 \qquad 2^2 > 1+1$

$\rightsquigarrow$ vortices <u>repel</u> each other and do not form bound states

$\underset{v}{\overset{k_1}{\circ}} \xleftarrow{\quad d \quad} \underset{v}{\overset{k_2}{\bullet}} \qquad E_{int} \sim \frac{k_1 k_2 n_s}{M} \ln R/d$

They behave like 2d point charges.

→ we will discuss it more later → Goson-vortex duality

4) Due to vortices even $T=0$ super-fluid can exhibit dissipation

Feynman 1955 Anderson 1966

→ quantum turbulence

5) By increasing temperature one can sometimes create an extensive number of vortices → BKT phase transition