Superconductivity – the Higgs mechanism and the fate of the Goldstone mode

Superconductor – charged superfluid

$$S = \underbrace{S_\theta}_{\text{matter}} + \underbrace{S_{EM}}_{\text{electro-magnetism}}$$

$$S_\theta = \int dt\, d^3x \, \frac{n_s}{2M^*}\left[\frac{1}{c_s^2}(D_t\theta)^2 - (D_i\theta)^2\right]$$

where $D_\mu\theta = \underbrace{\partial_\mu\theta - e^* A_\mu}_{\text{covariant derivative}}$ with $M^* = 2m$, $e^* = 2e$, $\hbar = 1$

$$S_{EM} = \frac{1}{2\mu}\int dt\, d^3x \left(\left(\frac{E}{c}\right)^2 - B^2\right)$$

$$E_i = \partial_t A_i - \partial_i A_t \qquad B_i = \epsilon_{ijk}\partial_j A_k$$

S is invariant under <u>local</u> $U(1)$ gauge transformations

$$\theta \to \theta + \alpha \qquad A_\mu \to A_\mu + e^*\partial_\mu\alpha$$

The Goldstone phase θ can now be <u>absorbed</u> into the gauge field A_μ via a gauge transf.

$$S = \int dt\, d^3x \left[\frac{1}{2}\frac{n_s}{M^*}e^{*2}\left\{\left(\frac{A_t}{c_s}\right)^2 - A_i^2\right\} + \frac{1}{2\mu}\left\{\left(\frac{E}{c}\right)^2 - B^2\right\}\right]$$

New length scale intrinsic to superconductors

London penetration length λ_L

$$\frac{1}{\mu_s \lambda_L^2} = \frac{n_s}{M^*} e^{*2} \Rightarrow \lambda_L = \sqrt{\frac{M^*}{n_s (e^*)^2 \mu}}$$

physical meaning will be clarified later

We found a theory of a massive U(1) gauge field ⇝ photon has a finite mass (short-ranged) ⇒ screening

This is called the Higgs mechanism

– massless Goldstone and photon disappeared

Introduce ρ and j^i from S_θ:

$$\rho \equiv - \frac{\delta S_\theta}{\delta A_t} = - \frac{1}{\mu c_s^2 \lambda_L^2} A_t$$

$$j^i \equiv - \frac{\delta S_\theta}{\delta A_i} = + \frac{1}{\mu \lambda_L^2} A^i \rightsquigarrow \text{London equation}$$

Now use it in EOM $\frac{\delta S}{\delta A_\mu} = 0$;

$$\frac{\delta S}{\delta A_t} = 0 \Rightarrow \text{Gauss law} \quad \epsilon \nabla \cdot E = \rho$$

$$\frac{\delta S}{\delta A_i} = 0 \Rightarrow \text{Ampere law}$$

$$\frac{1}{\mu} \left(\frac{1}{c^2} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} \right) = j^i$$

Write the Ampere law in terms of A_μ,

use the continuity equation $\nabla \cdot j + \partial_t \rho = 0$

$$\boxed{\left(\underbrace{\Box}_{-\frac{1}{c^2}\partial_t^2 + \nabla^2} \vec{A} - \frac{1}{\lambda_L^2}\vec{A}\right) = \left[1 - \left(\frac{c_s}{c}\right)^2\right] \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)}$$

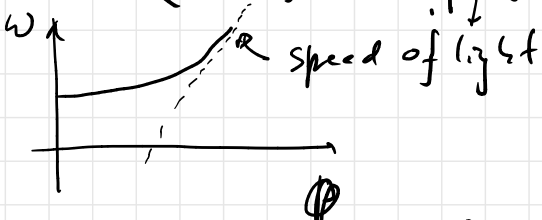
Solve in fourier space:

a) <u>transverse modes</u> $\vec{\nabla}\cdot\vec{A} = 0$

$\Downarrow$

RHS vanishes

$$\omega^2 = (m_A c^2)^2 + (qc)^2 \quad \text{with} \quad m_A = \frac{1}{\lambda_L c}$$



static case $(\partial_t = 0) \Rightarrow$ Meissner effect

$$\partial_x \vec{A}_T - \frac{1}{\lambda_L^2}\vec{A}_T = 0 \quad \Rightarrow \quad A_T \sim e^{-x/\lambda_L}$$

magnetic field is expelled from the superconductor

b) longitudinal modes $\vec{\nabla}\times\vec{A} = 0 \Rightarrow$ RHS almost cancels $\partial_t^2 A$ on the LHS

$$\omega^2 = (m_A c^2)^2 + (q c_s)^2$$

$c_s \rightarrow$ speed of sound

this is the plasmon mode which goes into metalic plasmon in the normal state $T > T_c$

In non-relativistic SC, ℓ- and t- modes have very different group velocity of propagation. In a Lorentz-invariant SC ($c_s = c$), the RHS of the Ampere law vanishes $\Rightarrow$ both ℓ- and t-modes propagate with the velocity of light

Superconductors are not covered in detail in this lecture, more in books P. Coleman, Tinkham, de Gennes

1) Type I vs Type II SC
$\left(\lambda_L < \frac{1}{\sqrt{2}} \xi\right)$ $\quad$ $\left(\lambda_L > \frac{1}{\sqrt{2}} \xi\right)$

depends on the surface energy of a domain wall SC–N, vortex crystals

2) Vortices are exponentially localized

$$\vartheta_{sc} \sim \exp\left(-r/\lambda_L\right)$$

and a magnetic flux $\Phi_V = \frac{h}{e^*} = \pi \frac{\hbar}{e}$ binds to every vortex