This is the most general structure for interacting spinless (either bosonic or fermionic model) in 1d. while in the relativistic TL model u and K can be determined exactly in terms of microscopics, in other int. models these parameters are difficult to compute (can be measured experimentally)

Correlation functions and thermodynamics

Euclidean

ϕ, θ - dimensionless angles

Since the action is quadratic in ϕ, we can extract $\langle \phi\phi \rangle$ from it.

First, introduce $y = u\tau$

$$S = \frac{1}{2\pi K} \int dx\, dy \left((\partial_y \phi)^2 + (\partial_x \phi)^2 \right)$$

in Fourier space $x, y \sim \vec{q}$

$$S = \frac{1}{2\pi K} \int \frac{d^2 q}{(2\pi)^2} \, \phi(-\bar{q}) \; q^2 \; \phi(\bar{q})$$

and thus $\quad G^{-1} \sim K^{-1} q^2$

$$G(\bar{q}) \sim K \, q^{-2} \;\Rightarrow\; G(x,y) \sim K \log \frac{x^2+y^2}{\ell^2}$$

ℓ – some length scale?

$$\sim K \log \frac{x^2+u^2\tau^2}{\ell^2}$$

For $\langle \theta\theta \rangle$ and $\langle \theta\varphi \rangle$ correlators, see Giamarchi book. Using these, single particle Green's function:

$r = (x, u\tau)$, $\alpha \to 0_+$

$$G_R(r) = -\langle \psi_R(r) \, \psi_R^\dagger(0) \rangle$$

$$= -\frac{e^{ik_F x}}{2\pi\alpha} \langle e^{i(\varphi(r)-\theta(r))} \, e^{-i(\varphi(0)-\theta(0))} \rangle$$

$$= -\frac{e^{ik_F x}}{2\pi\alpha} \, e^{-\left(\frac{K+K^{-1}}{2} F_1(r) + F_2(r)\right)}$$

$F_1(r)$: $\frac{1}{2} \log\left(\frac{x^2 + (u|\tau| + \alpha)^2}{\alpha^2}\right)$

$F_2(r)$: $i \arg(y_\alpha + ix)$

for $K=1$, Euclidean

$$G_R(r) = \frac{-i e^{ik_F x}}{2\pi} \, \frac{1}{x + i(v_F\tau + \alpha \, \mathrm{sgn}(\tau))}$$

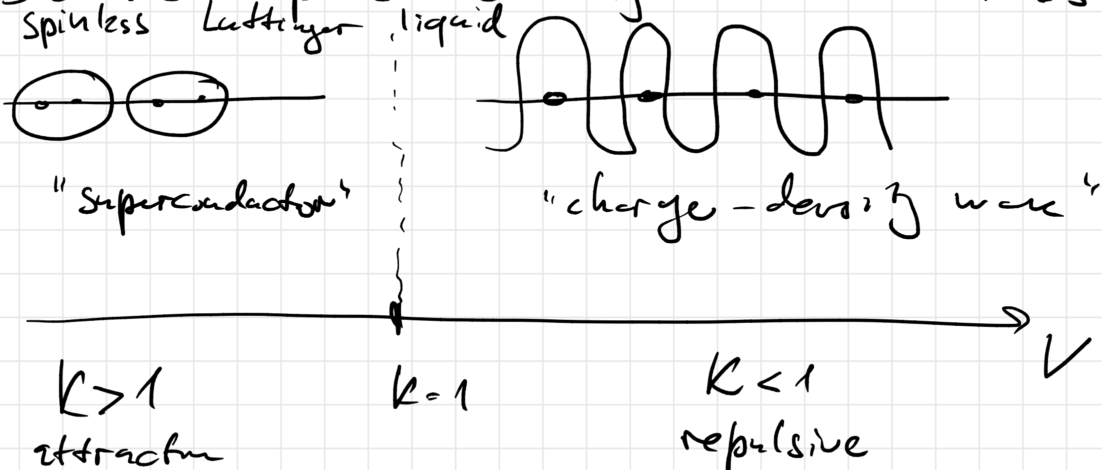
after performing the Wick rotation

$$\tau = it + \alpha\,\mathrm{sgn}\,t$$

we find that the denominator equals to zero for $x - v_F t = 0$
(right-moving particle)

thus $K=1$ → free problem

In the presence of interactions



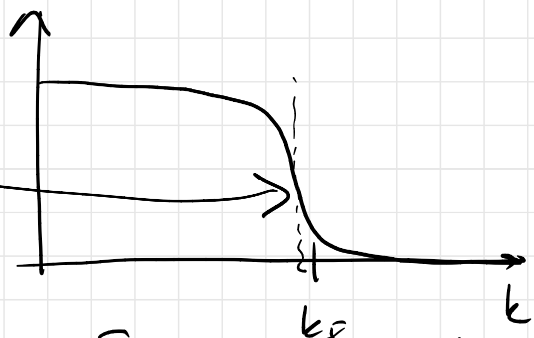
In 1d no spontaneous symmetry breaking of continuous symmetry (OLRO) due to diverging IR fluctuations (Mermin-Wagner theorem)

Since $\frac{K+K^{-1}}{2} \geq 1$, in the presence of interactions, G_R decays faster than in a free system. This has an important consequence:

$$n(k) = \int dx \, e^{-ikx} \, G_R(x, \tau = 0^-)$$

$$e^{ik_F x} \left(\frac{1}{x}\right)^{\frac{K+K^{-1}}{2}}$$

$$\sim |k - k_F|^{\frac{K+K^{-1}}{2} - 1}$$



Contrary to Landau Fermi liquid, $n(k)$ has <u>no</u> discontinuity at the Fermi momentum. Single-particle fermionic excitations $\rightsquigarrow$ collective bosonic mode in 1d.

Single-particle density of states <u>vanishes</u> at $E \simeq E_F$

Adding spin degrees of freedom $\rightsquigarrow$ rich physics in 1d, see Giamarchi book.