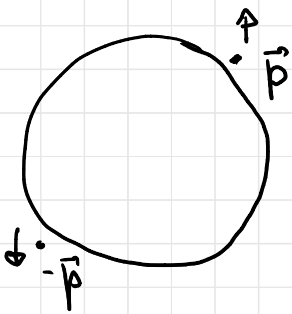
Chiral pairing

In conventional superconductors —

S-wave spin singlet pairing



*) Cooper pair has vanishing spin

*) The gap $\Delta = \langle \psi_{\uparrow p} \psi_{\downarrow -p} \rangle$ is constant on the Fermi surface

There are however more exotic types of pairing — first discovered in superfluid ^3He in 1970s $T_c \sim 10^{-3}\,K$

chiral p-wave spin-triplet pairing

Consider a model many-body systems of spinless fermions (e.g. polarized by magnetic field or single-component cold atoms) with attractive short-range interactions

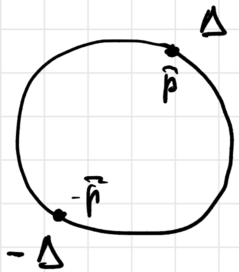
How are these fermions paired?

$$\Delta = - V(\vec r_1 - \vec r_2) \langle \hat\psi(\vec r_1) \, \hat\psi(\vec r_2) \rangle$$

due to the Pauli principle cannot set $\vec r_1 = \vec r_2$

in a homogeneous system Δ depends on $\vec r_1 - \vec r_2$

in momentum space Δ is not constant on the Fermi surface



$$\Delta_{\vec p} \sim \langle \hat\psi_{\vec p} \, \hat\psi_{-\vec p} \rangle$$

using $\{\psi_p, \psi_q\} = 0$

$$\Delta_{\vec p} = - \Delta_{\vec p}$$

in 3d: Δ will have nodes (vanish) at certain points of the Fermi surface

in 2d: can do better, completely gapped chiral state $\Delta_{\vec p} = (p_x \pm i p_y) \hat\Delta_{|\vec p|}$ const on the FS

a) winds around the Fermi surface

b) $\Delta_{\vec p} = - \Delta_{-\vec p}$

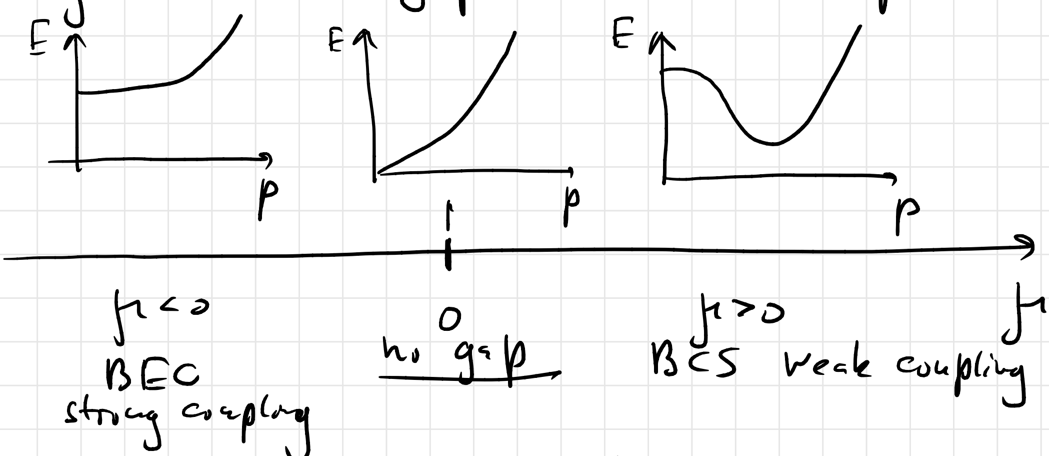
c) breaks time-reversal and parity symmetries

BCS mean-field treatment: Read-Green 2000

$$H_{BdG} = \begin{pmatrix} \epsilon_p & \Delta_{\bar{p}} \\ \Delta^*_{-\bar{p}} & -\epsilon_p \end{pmatrix} \qquad \epsilon_p = \frac{p^2}{2m} - \mu$$

$$E_p = \pm\sqrt{\epsilon_p^2 + |\Delta_p|^2} = \pm\sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + p^2|\hat{\Delta}|^2}$$

at $\mu = 0$ the gap closes at $p = 0$



$\mu = 0$ is a quantum phase transition

Can the BEC and BCS regimes be different phases of matter despite having the same type of the Landau order parameter?

Remember Anderson pseudospin picture?

$$H_{BdG} = \vec{h}(\bar{p}) \cdot \vec{\tau}$$

$$h(\bar{p}) = (\mathrm{Re}\,\Delta_p, -\mathrm{Im}\,\Delta_p, \epsilon_p) = (p_x\hat{\Delta}, -p_y\hat{\Delta}, \epsilon_p)$$

if $\mu \neq 0$, $\vec{h}(\bar{p}) \neq 0$ and we can define a unit vector $\hat{h}(p) = \vec{h}(\bar{p}) / |\vec{h}(\bar{p})| \in S^2$

Since as $p \to \infty$ $\hat{h}(p) \to (0,0,1)$, the Hamiltonian defines a mapping

$$\underset{p\text{-space}}{S^2} \longrightarrow \underset{\hat{h}\text{-space}}{S^2}$$

Some mappings of this type cannot be smoothly deformed into each other can be distinguished by the <u>topological invariant</u> known as <u>Chern number</u>

$$C = \int \frac{d^2p}{4\pi} \underbrace{\hat{h} \cdot \left(\partial_{p_x} \hat{h} \times \partial_{p_y} \hat{h}\right)}_{\text{solid angle swiped by } \hat{h} \text{ as element of momentum space is traversed}}$$

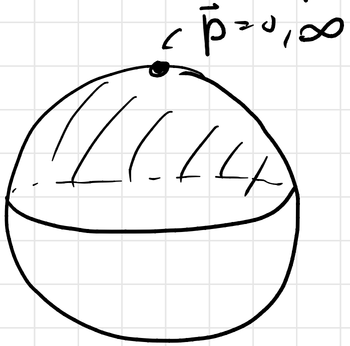
*) C is an <u>integer</u>

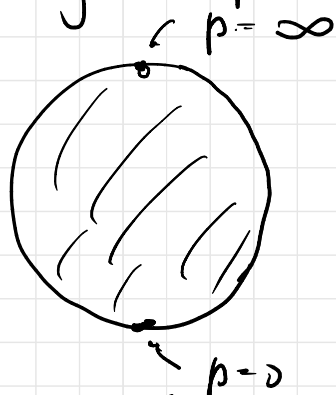
*) C is <u>invariant</u> under smooth changes of $\hat{h}$ – defines a whole phase

*) C is the <u>skyrmion number</u> of the Anderson magnetic field

we plot now S^2 of $\hat{n}$:

at fixed $|\vec{p}|$, $\hat{n}_x$ and $\hat{n}_y$ sweep a circle





$\mu<0$ BEC

$C=0$

topologically trivial phase

$\mu>0$ BCS

$C=1$

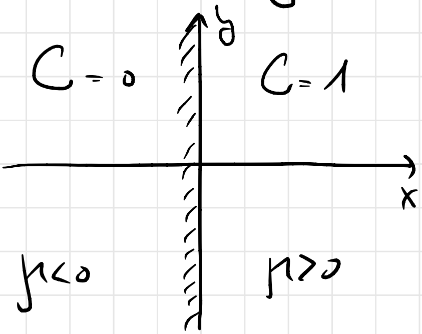
topologically nontrivial phase

→ Chern number distinguishes the two phases

<u>Index theorem:</u> $\mu=0$ → topological phase transition

Interfaces of topologically distinct phases host <u>chiral fermionic modes</u> <u>localized</u> (bound) at the <u>boundary</u>

Their algebraic number $\nu = \Delta C$



$\mu(x)$ – monotoneous increasing function of x

$\mu(x)=0$

$$H_{BdG}\,\Psi = E_{BdG}\,\Psi$$

$p+ip$ SC

Ansatz: $\Psi = e^{ik_y \cdot y}\begin{pmatrix} u(x) \\ v(x) \end{pmatrix}$ translation inv. along y-direction

First set $\underline{k_y = 0}$ and look for $E=0$ solution:

neglecting second derivatives $\rightsquigarrow$ <u>Dirac-like equation</u>

no loss of generality $\hat{\Delta} > 0$

$$i\partial_t u = -\mu u - i\hat{\Delta}(\partial_x + i\partial_y) v$$
$$i\partial_t v = \mu v - i\hat{\Delta}(\partial_x - i\partial_y) u$$

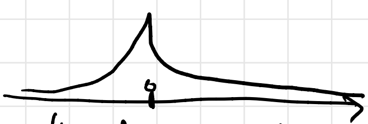
These equations are compatible with the <u>Majorana</u> (reality) condition $u = v^*$

At $E = 0$ we solve

$$-\mu u - i\hat{\Delta}\partial_x v = 0$$
$$\mu v - i\hat{\Delta}\partial_x u = 0$$

$$\Psi = \begin{pmatrix} 1 \\ -i \end{pmatrix} f(x) \Rightarrow \left(-\mu(x) - \hat{\Delta}\partial_x\right) f = 0$$

$$\leftarrow f(x) \sim \exp\left(-\frac{1}{\hat{\Delta}}\int^x \mu(x)\, dx\right)$$



localized near $x=0$

if we normalize $N = e^{i\pi/4} \rightarrow$ <u>Majorana ($u = v^*$)</u>

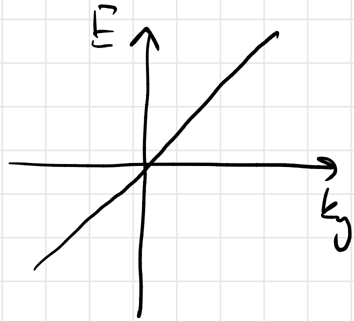
<u>Bound</u> state at $E = 0$.

Consider now $\underline{k_y \neq 0}$

$$E u = -\mu u - i\hat{\Delta}\partial_x v + i\hat{\Delta} k_y v$$
$$E v = +\mu v - i\hat{\Delta}\partial_x u - i\hat{\Delta} k_y u$$

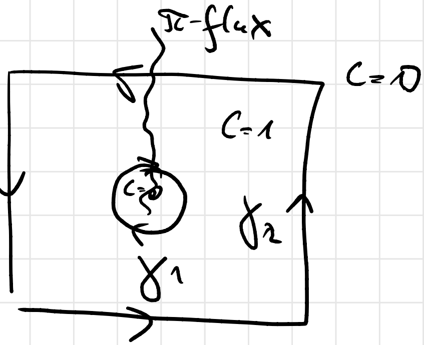
treat now E and k_y as perturbations to the $E=0$ solution:

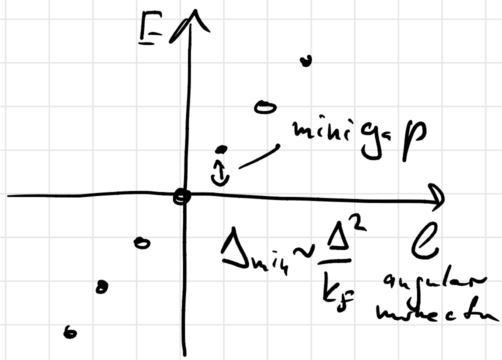
$$E = \hat{\Delta} k_y \qquad v = \frac{\partial E}{\partial k_y} = \hat{\Delta} > 0$$



<u>chiral</u> solution – propagates only in one direction

If we view a vortex in p+ip topological superconductor as an interface ($C=0$ vs $C=1$)





<u>Majorana fermions</u> localized on vortices have <u>non-abelian</u> quantum statistics

$\Downarrow$

topological quantum computer

J. Alicea review 2012